# Does the quark-gluon plasma contain stable hadronic bubbles?

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## Abstract

We calculate the thermodynamic potential of bubbles of hadrons embedded in quark-gluon plasma, and of droplets of quark-gluon plasma embedded in hadron phase. This is a generalization of our previous results to the case of non-zero chemical potentials. As in the zero chemical potential case, we find that a quark-gluon plasma in thermodynamic equilibrium may contain stable bubbles of hadrons of radius $R \simeq 1$ fm. The calculations are performed within the MIT Bag model, using an improved multiple reflection expansion. The results are of relevance for neutron star phenomenology and for ultrarelativistic heavy ion collisions.

12.38.Mh, 12.39.Ba, 97.60.Jd, 98.80.Cq

## I. INTRODUCTION

In a previous paper [1] we calculated the free energy of a bubble of hadrons embedded in an extended quark-gluon plasma (QGP), and of a droplet of QGP embedded in an extended hadron phase, for parameters in the vicinity of the cosmological quark-hadron transition, i.e. the baryon chemical potential were set to zero. In Ref. [1] we found, as Mardor and Svetitsky in [2], that the free energy of a hadron bubble of radius $R$ embedded in QGP possessed a minimum at radii of a few fm, even above the phase transition temperature. Thus, hadronization from QGP is strongly enhanced compared to the usual nucleation scenario, where an energy barrier has to be overcome before bubbles of hadrons can form and grow.

An important ingredient in these calculations is the density of states of the relevant particles. In Ref. [1] we advised a modification of the usual expressions arising from the multiple reflection expansion (MRE), and we saw that this modification (the MMRE) yielded more accurately the free energy than the MRE did when compared to a direct (but numerically heavier) sum-over-discrete-states calculation. Our modification of the multiple reflection expansion expressions is physically well motivated, in that it consists of a truncation of the density of states in such a way that we avoid the effects of a negative density of states, present in the usual MRE expressions.

In this paper, we generalize the calculations to finite chemical potential, and we shall see that also in the case of non-zero chemical potential, the MMRE produces accurately the thermodynamic potential. We show that, as in the case of zero chemical potential, the thermodynamic potential of a hadron bubble embedded in QGP has a minimum at a radius of $R \simeq 1$ fm, meaning that it is thermodynamically favorable for a QGP in equilibrium to spontaneously create bubbles of hadrons of about this size. However, the minimum in the thermodynamic potential found for non-zero chemical potential is less pronounced than in the zero chemical potential case considered in Ref. [1]. The more general calculations presented in this paper should be relevant also to neutron star phenomenology as well as ultrarelativistic heavy ion collisions.

In the following section we give an overview of the basic theory leading to the results of section III. Finally in Sec. IV, we summarize the conclusions.

## II. THEORETICAL FRAMEWORK

### A. The particle model

#### *1. The quark-gluon plasma*

The model of the plasma phase is a close derivative of the MIT Bag model [3,4]. It consists essentially of non-interacting quarks and gluons (also self-interaction of the gluons is neglected), bounded by MIT Bag-like boundary conditions [5]. There are three quark flavors in our model: $u$ and $d$, which we consider massless, and $s$, to which we assign a mass of $m_s = 150$ MeV. The other quarks are too heavy to be relevant in this analysis. We choose a critical temperature (the bulk phase equilibrium temperature at zero chemical potential) of $T = 150$ MeV, which gives a Bag constant of $B = (221\ \mathrm{MeV})^4$. The plasma may be inside a sphere, as in the usual MIT Bag (we shall refer to this configuration as a plasma droplet),

or outside, i.e. in a "vacuum bubble" configuration. For a detailed review of this model, the reader may consult [1]. We are mostly interested in the vacuum bubble situation, where the vacuum bubble is filled with a thermal mixture of hadrons as described later.

For the density of states of the quarks and gluons we use a modification of the multiple reflection expansion (MRE) [6–10], advised in [1]. This modification affects only the gluon density of states, which is then

$$\rho_g(k,R) = \begin{cases} 0, & 0 \le kR < 0.832 \\ \frac{V_{QGP}k^2}{2\pi^2} - \frac{4R}{3\pi}, & kR \ge 0.832 \end{cases} \tag{1}$$

for gluons inside a sphere (i.e. the MMRE($R$)), and

$$\rho_g(k,R) = \begin{cases} 0, & 0 \le kR < 0.458 \\ \frac{-V_H k^2}{2\pi^2} + \frac{4R}{3\pi}, & kR \ge 0.458 \end{cases} \tag{2}$$

for the difference between the gluon density of states in a volume $V_\infty$ and in volume $V_\infty - V_H$, (i.e. the MMRE($-R$)).

In [1] we saw that this modified multiple reflection expansion (MMRE) description of the density of states made the free energy agree nicely with more direct calculations, i.e. summing over discrete energy levels. Before using the MMRE in the more general case under consideration in this paper, we performed similar checks of the MMRE against direct sum-over-discrete-states calculations. Also in this case, the MMRE proved to describe the density of states adequately and significantly better than the MRE, cf. Fig. 1. For a complete description of the MMRE density of states, we refer the reader to [1].

#### *2. The hadron phase*

We include all hadrons with masses below 1.2 GeV, taking only the volume part of the density of states into account. This is justified as the hadrons represent far fewer degrees of freedom than the QGP. Specifically, the density of states of a hadron species occupying a volume $V_H = \frac{4\pi}{3}R^3$ is

$$\rho_H(k,R) = \frac{V_H k^2}{2\pi^2}. \tag{3}$$

### B. Thermodynamics

The thermodynamic potential is

$$\Omega(T,V,\{\mu_i\}) = -T\ln(Z(T,V,\{\mu_i\})), \tag{4}$$

where $Z(T,V,\{\mu_i\})$ is the partition function

$$Z(T,V,\{\mu_i\}) = Tr\left\{e^{-(\mathcal{H}-\sum_i \mu_i\Lambda_i)/T}\right\}. \tag{5}$$

Here, $\mathcal{H}$ is the Hamiltonian, the $\Lambda_i$'s are the symmetries of the Hamiltonian (the conserved quantities), and $\{\mu_i\}$ denotes the corresponding collection of chemical potentials. $Tr\{..\}$ means the sum over all diagonal elements of energy- and number-eigenstates. These are simultaneous eigenstates since $[\mathcal{H}, \Lambda_i] = 0$. The volume dependence enters via the energy levels, i.e. in the trace operation.

In thermodynamic equilibrium, the configuration realized in Nature is the one which minimizes the thermodynamic potential, leading to the Gibbs conditions for phase equilibrium: $T_1 = T_2$ (thermal equilibrium), $-\frac{\partial \Omega_1}{\partial V_1} = -\frac{\partial \Omega_2}{\partial V_2}$ (mechanical equilibrium), and $\mu_i^{(1)} = \mu_i^{(2)}$ (chemical equilibrium), where index 1 and 2 refer to the two phases.

If there are no interactions between the particles, i.e. the energy of a particular state is the sum of the energies of the individual particles, then we can write the thermodynamic potential for a particular particle species enclosed in a spherical volume of radius $R$, as

$$\Omega(R, T, \mu) = \mp g T \int_0^\infty dk\, \rho(k, R) \ln(1 \pm e^{-(\sqrt{k^2+m^2}-\mu)/T}), \tag{6}$$

(upper sign for fermions, lower sign for bosons) where we further assume that the energy levels are sufficiently closely spaced that we may use a smoothed density of states, $\rho(k, R)$, (e.g. the MMRE) normalized such that $\int_0^\infty dk\, \rho(k, R)$ is the total number of states in the volume $V = \frac{4\pi}{3} R^3$. The chemical potential entering in (6), is the combined chemical potential of that particle species

$$\mu = \sum_i \lambda_i \mu_i, \tag{7}$$

$\lambda_i$ being the quantum expectation value of the symmetry operator $\Lambda_i$ of the particle species in question (e.g. $\lambda_{\text{baryon}} = 1/3$ for a quark). Finally, $g$ is an appropriate degeneracy factor.

At the energies relevant in this analysis, the quarks and gluons can be considered point-like. This is not the case for the hadrons. We must take explicit account of the fact that the hadrons occupy a certain volume. A number of such excluded volume corrections have been discussed in the literature, but we shall not go into an analysis of the different suggestions. We choose the one proposed in [11], since it is very easy to implement and grasps much of the essential physics involved. According to [11], the "true" pressure of the hadron gas ($p_H$) is obtained from the ideal gas pressure ($p_{H,id}$) according to

$$p_H = \frac{p_{H,id}}{1 + \epsilon_{H,id}/(4B)}, \tag{8}$$

where

$$\epsilon_{H,id} = \sum_i \epsilon_{i,id} \tag{9}$$

is the total energy density of the ideal hadron gas,

$$p_{H,id} = \sum_i p_{i,id} \tag{10}$$

is the total hadronic ideal gas pressure, ($i \in$ hadron species), and $B$ is the Bag constant. We note that some sort of excluded volume correction is essential, since in a point-like hadron gas, no transition to QGP will occur at low temperatures, even at arbitrarily high density.

There is one more question about the hadrons we need to address before we proceed. This is the phenomenon of Bose-Einstein condensation, regarding the bosonic part of the hadron gas. A bosonic gas always has $\mu_i \leq m_i$, and when $\mu_i = m_i$ particles will start to "condense" into the lowest, zero-momentum state, meaning that there will be a macroscopic number of particles in this state. When the parameters are such that we would have $\mu_i \geq m_i$, the hadron species $i$ is removed from the above summations (9) and (10), since particles in a zero-momentum state do not contribute any pressure.

## III. RESULTS

We shall consider both the case of equal chemical potentials, $\mu_u = \mu_d = \mu_s \equiv \mu_q$, and the case where the $u$ and $d$ quarks have a different chemical potential from that of the $s$ quark, $\mu_u = \mu_d \neq \mu_s$. The first case is relevant when the time scales are such that the weak interactions maintain equilibrium between all three flavors of quarks (e.g. quark matter in neutron stars, neglecting the electron chemical potential as a first approximation), whereas the case of separate $s$ quark chemical potential is relevant to ultrarelativistic heavy ion collisions, the time scales here being such that the net number of $s$ quarks is conserved separately from that of the net number of light quarks. In our model, the two light quarks are both massless, and since we neglect electromagnetic interactions, there is no difference between $u$ and $d$ quarks.

### A. Quark-hadron phase equilibrium

To study the phase equilibrium between the hadrons and the QGP, we implement the Gibbs conditions on the volume (or bulk) part of the thermodynamic potential, i.e. also in the QGP density of states we only use the terms proportional to the volume. (We shall see later that the surface terms give rise to interesting features near bulk phase equilibrium.) In this way, we obtain the phase diagram in Fig. 2. The QGP phase (above the phase equilibrium lines) occupies a larger and larger region of the phase diagram as the $s$-quark chemical potential is raised. As one might have expected, the phase equilibrium line in the case of equal chemical potentials lies somewhere between the lines of varying $\mu_s$.

### B. The thermodynamic potential

In this section we look at the thermodynamic potential in the vicinity of equilibrium, now using the full expression of the density of states of the QGP (i.e. including the surface terms). In Figs. 3–6, there is a single chemical potential, $\mu_q$, common to all three quark flavors.

First we show Fig. 3, the thermodynamic potential of a QGP droplet in equilibrium with an extended hadron phase near the bulk equilibrium point $(\mu_q, T) = (320, 106.1)$MeV. The picture is as expected for a first order phase transition: Below the bulk equilibrium temperature no stable QGP droplet can form, and even somewhat above this temperature there is an energy barrier for the system to pass before the true minimum at $R = \infty$ can be reached. The surface terms are responsible for this energy barrier. Precisely *at* the phase

transition temperature the volume terms cancel (by definition), and the thermodynamic potential goes to infinity as $R^2$ (the leading surface term).

The more interesting conclusions are reached when one considers the reverse situation, namely a hadron bubble in equilibrium with an extended phase of QGP. Figs. 4, 5 and 6 show the thermodynamic potential of this configuration near different points on the phase equilibrium line. In all three cases the potential exhibits a minimum at a radius of $R \simeq 1$ fm of approximately the same depth, even at temperatures above phase equilibrium temperature. Such a minimum would not be present in the standard textbook treatment of phase transitions, in which a first order phase transition usually is described in terms of a phenomenological free energy containing only volume- and (positive) surface terms.

Thus, a QGP in thermodynamic equilibrium apparently contains bubbles of hadrons, and the transition from QGP to hadrons will proceed (in addition to ordinary bubble nucleation and subsequent expansion of these bubbles) by smooth expansion of the pre-formed bubbles. In addition to this smooth growth as the minimum tends to larger radii when the temperature drops, pre-formed bubbles can also grow discontinuously by passing the barrier from $R > 0$ to some $R$ outside the barrier. This possibility resembles very much the ordinary nucleation scenario with a modified barrier height. At the parameters of Figs. 4, 5 and 6, the mean number of quarks in a hadron bubble of radius $R = 1$ fm is typically 3–4, suggesting that formation of a real hadron is not a very rare event. However, the minimum being of modest depth, this interesting effect could be due to the model's inaccurate representation of QCD.

The fact that there is no energy barrier for a hadron bubble to form in QGP does not imply that the phase transition is second order, or a smooth cross-over. In the model adopted here, the transition is inherently first order, since there is an *a priori* difference in entropy between the two bulk phases, and thus a latent heat. As seen from e.g. Fig. 4 it is not energetically favorable for bulk hadronic matter to form at temperatures above the phase transition temperature $T_0$. But the formation of isolated bubbles of hadronic matter *is* favorable above $T_0$, so the transition will appear smoother than normally expected for a first order transition, with a more gradual release of latent heat.

We now investigate the effect of letting the $s$ quark have its own chemical potential. To this end we show Figs. 7 and 8. In Fig. 7 the $s$ quark chemical potential is 4 times greater than the light quark chemical potential, whereas in Fig. 8 it is converse. The conclusion to be drawn from these figures is that a large $s$ quark chemical potential tends to wash out the minimum in the thermodynamic potential, present when the light quark chemical potential is less than or equal to the $s$ quark chemical potential. This behavior can be traced back to the positive surface tension contribution from the massive $s$-quarks to the thermodynamic potential.

Finally, we underline that the interesting effect of bubble formation is due to the fact that the presence of a surface alters the density of states of particles. Although these finite-size effects are negligible at large radii of the bubble or droplet, they may have important implications for the phase transition as a whole.

## IV. CONCLUSION

We have seen that the thermodynamic potential of a hadron bubble embedded in quark-gluon plasma, exhibits a minimum at a radius $R \simeq 1$ fm, even at temperatures somewhat

above the bulk transition temperature. Thus, within the model described here, a homogeneous plasma of size larger than a few fermi is an impossibility.

Regarding the relevance of these results in connection with current and forthcoming ultrarelativistic heavy ion collisions, we conclude that *if* indeed a quark-gluon plasma is formed in the course of such a collision, then (according to the model considered here) this plasma phase will contain bubbles of hadrons of radius $R \simeq 1$ fm. *If* such hadronic bubbles form inside the plasma, the observable effects are likely to include some blurring of the plasma signatures, since this reduces the effective plasma volume.

We emphasize that these conclusions may well be model dependent, inasmuch the minimum of the thermodynamic potential is of modest depth. However, the phenomenon of spontaneous creation of stable hadronic bubbles in a quark-gluon plasma, does seem to be well established within the the model discussed here [1,2,12]. The main question is now, whether these hadronic bubbles are of physical nature, or merely an artifact of the model. Certainly, it would improve confidence in these results if other models of QCD, and eventually lattice calculations, were to yield similar results.

## ACKNOWLEDGMENTS

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FIGURES

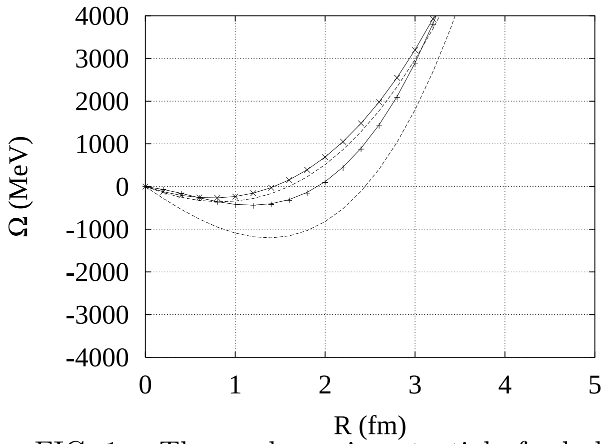


FIG. 1. Thermodynamic potential of a bubble of hadron phase of radius $R$ embedded in an extended QGP phase. In the two upper curves $(\mu_q, T) = (450, 46)$ MeV, i.e. a higher temperature than the phase transition temperature at this chemical potential, $T_0 = 41.0$ MeV. In the two lower curves $(\mu_q, T) = (200, 139)$ MeV, again slightly above the phase transition temperature $T_0 = 133.7$ MeV. To draw the solid lines, the MMRE($-R$) has been used to describe the density of states in the expression for the thermodynamic potential, whereas the dashed lines are for the MRE($-R$). Based on comparisons with more direct sum-over-states like calculations (the points practically coinciding with the MMRE($-R$) lines), we conclude that the MMRE($-R$) is the more correct model for the density of states.

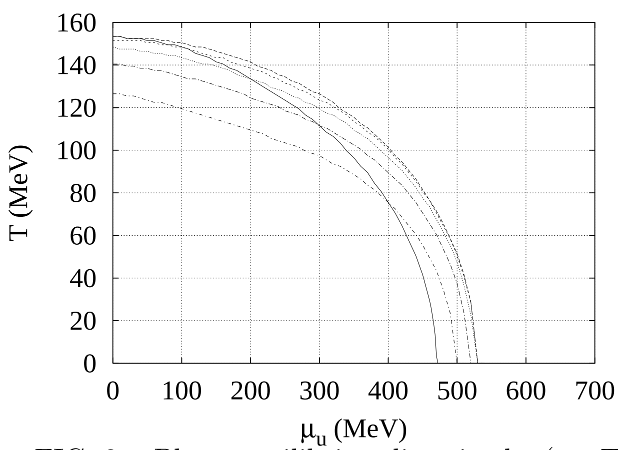


FIG. 2. Phase equilibrium lines in the $(\mu_u, T)$-plane. At these lines, QGP and hadrons are in thermodynamic equilibrium. The solid line represents the case of equal quark chemical potentials, $\mu_u = \mu_d = \mu_s$, whereas the other lines represent different values of the $s$-quark chemical potential: Top to bottom, $\mu_s = 0, 100, 200, 300, 400$ MeV.

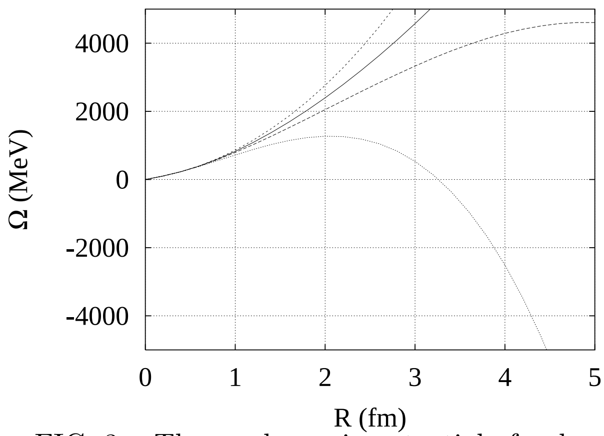


FIG. 3. Thermodynamic potential of a droplet of QGP of radius $R$ embedded in an extended phase of hadrons, normalized such that the thermodynamic potential of a pure hadron phase is zero. The plasma density of states is described by the MMRE($R$). The bulk phase transition point is $(\mu_q, T) = (320, 106.1)$ MeV (cf. Fig. 2). Curves are shown for different temperatures, top to bottom: $T = 104, 106.1, 108, 112$ MeV. The chemical potential is fixed at $\mu_q = 320$ MeV.

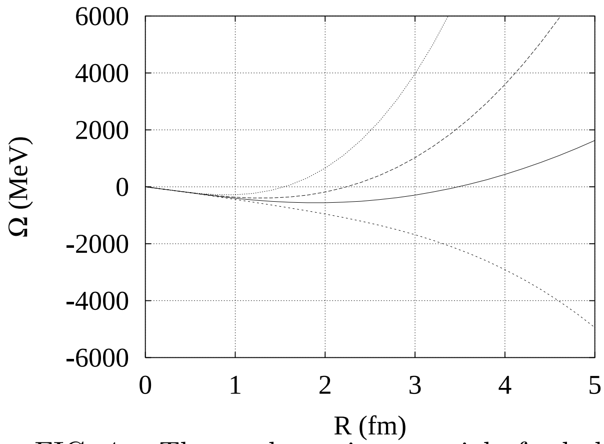


FIG. 4. Thermodynamic potential of a bubble of hadron phase of radius $R$ embedded in an extended QGP phase, normalized such that the thermodynamic potential of a pure QGP phase is zero. The plasma density of states is described by the MMRE($-R$). The bulk phase transition point is $(\mu_q, T) = (320, 106.1)$ MeV (cf. Fig. 2). The chemical potential is fixed at $\mu_q = 320$ MeV, and the temperature varies, top to bottom: $T = 112, 108, 106.1, 104$ MeV.

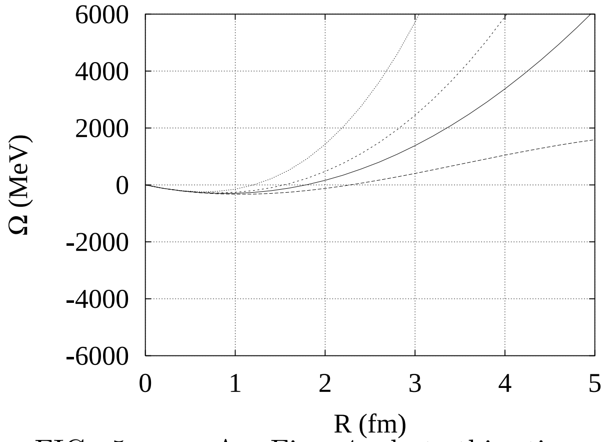


FIG. 5. As Fig. 4, but this time investigating the bulk phase transition point $(\mu_q, T) = (450, 41.0)$ MeV. The chemical potential is fixed at this value, and the temperatures are, top to bottom: $T = 52, 44, 41.0, 38$ MeV.

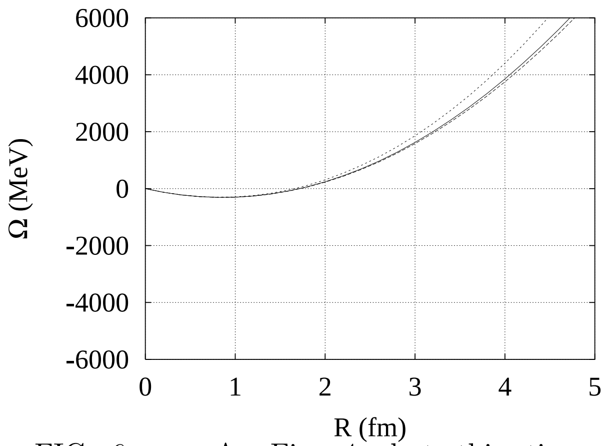


FIG. 6. As Fig. 4, but this time investigating the bulk phase transition point $(\mu_q, T) = (470.0, 3.28)$ MeV. The chemical potential is fixed at this value, and the temperatures are, top to bottom: $T = 8, 3.28, 1$ MeV. Notice that the depth of the minimum at $R \simeq 1$ fm is practically independent of which equilibrium point we consider, cf. Figs. 4 and 5. The curves in this figure differ somewhat in their qualitative behaviour from those in Figs. 4 and 5. This is because at these low temperatures, the surface terms dominate over the volume terms at the radii shown here; at larger radii the $T = 1$ MeV curve bends over and eventually tends to $-\infty$ as it should.

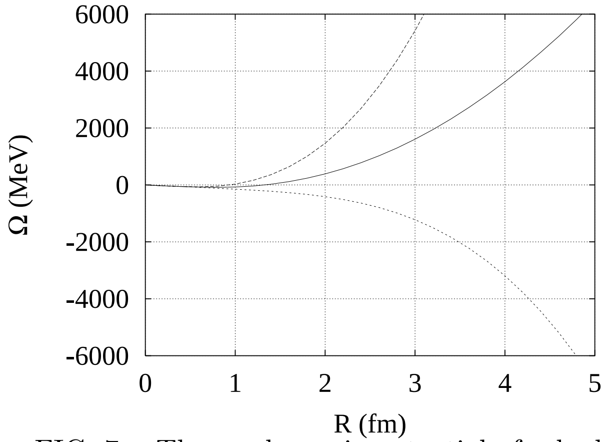


FIG. 7. Thermodynamic potential of a hadron bubble embedded in an extended QGP phase in the vicinity of the phase equilibrium point $\mu_u = 100$ MeV, $\mu_s = 400$ MeV, $T_0 = 119.5$ MeV (cf. Fig. 2). The chemical potentials are fixed at these values, and the temperature varies, top to bottom: $T = 125, 119.5, 115$ MeV. Notice that, at these values of the chemical potentials, the minimum in the thermodynamic potential practically disappears.

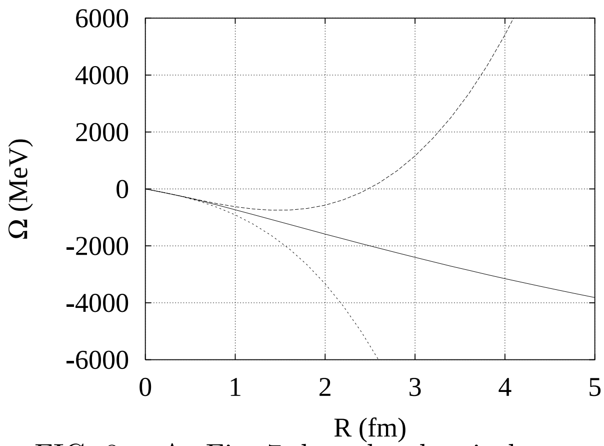


FIG. 8. As Fig. 7, but the chemical potentials are now $\mu_u = 400$ MeV $\mu_s = 100$ MeV, and the phase equilibrium temperature corresponding to these values is $T_0 = 100.0$ MeV. Again the chemical potentials are fixed, and the temperatures are, top to bottom: $T = 105, 100, 90$ MeV. The minimum in the thermodynamic potential is quite deep at these values of the parameters.